

ESTIMATING THE COEFFICIENT OF THERMAL CONDUCTIVITY OF ORDERED TWO-PHASE SYSTEMS

S. V. Stepanov

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Analysis of the relationships obtained in [1] for estimating the coefficient of thermal conductivity of two-phase systems shows that for isotropic systems they correspond with the known evaluations

$$\left[\frac{1-P}{\lambda^{(1)}} + \frac{P}{\lambda^{(2)}} \right]^{-1} \leq \lambda \leq \lambda^{(1)} (1-P) + \lambda^{(2)} P. \quad (1)$$

Different results from these are obtained for media which are statistically anisotropic. Hence the following relationships will be correct for fully ordered structures:

$$\frac{\lambda}{\lambda^{(1)}} \leq \left[1 - \frac{L_2}{L} + \frac{1}{L} \int_{(L_2)} \frac{dz}{1 + \frac{s_2(z)}{S} \left(\frac{\lambda^{(2)}}{\lambda^{(1)}} - 1 \right)} \right]^{-1}, \quad (2)$$

$$\frac{\lambda}{\lambda^{(1)}} \geq 1 - \frac{S_2}{S} + \frac{1}{S} \int_{(S_2)} \left[1 + \frac{l_2(x, y)}{L} \left(\frac{\lambda^{(1)}}{\lambda^{(2)}} - 1 \right) \right]^{-1} dx dy. \quad (3)$$

It is possible to show that the relationships (2) and (3) will give solutions which correspond with the solutions obtained by approximate methods of "linear isotherms" and of "linear thermal flow." From the physical point of view this is explained as follows. In the method of "linear isotherms" the actual temperature field in the system is replaced by a field in which the isotherms represent the combination of planes which are perpendicular to the macroscopic temperature gradient. Such a replacement is equivalent to the introduction of a number of infinitely thin superconducting planes into the system, which can only lead to an increase of the thermal conductivity.

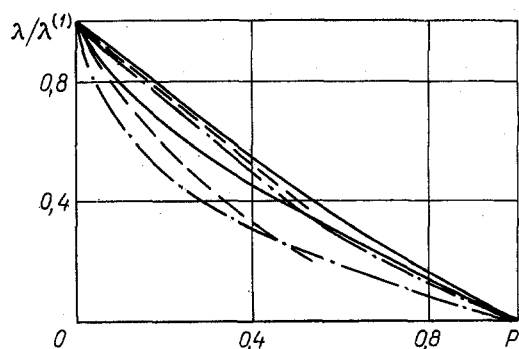


Fig. 1. Evaluation of the coefficient of thermal conductivity of ordered two-phase systems ($\lambda^{(2)}/\lambda^{(1)} = 0$) from above and below: continuous lines - cubes in a cubic pattern; dash lines - spheres in a cubic pattern; dot-and-dash line - Dul'nev-Frey model.

In the method of "linear thermal flow" the thermal flow lines are replaced by parallel straight lines. This corresponds with the introduction of a combination of tubes with heat-impervious walls of infinitely small thickness into the systems, which can only reduce λ .

Figure 1 gives the results of calculation of the thermal conductivity coefficient according to formulas (2) and (3) for the structures: cubes in a cubic pattern, spheres in a cubic pattern, a structure with mutually penetrating phases (Dul'nev-Frey model [2, 3]). Comparison of these with the solutions obtained by the methods of "linear isotherms" and "linear thermal flow" [2-6] shows complete agreement of the results.

NOTATION

- $\lambda^{(1)}, \lambda^{(2)}$ are the coefficients of thermal conductivity of the first and second phases;
- P is the volume concentration of the second phase;

High Temperatures Institute, Academy of Sciences of the USSR, Moscow. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 21, No. 1, pp. 181-182, July, 1971. Original article submitted June 5, 1970; abstract submitted December 21, 1970.

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L, S are the length and area of the foundation of a straight parallelepiped, selected as an element (the parallelepiped is orientated in the direction of the macroscopic temperature gradient);
 L_2 is the length of the projection of the second phase on the edge of the parallelepiped;
 S_2 is the area of the projection of the second phase on the foundation of the parallelepiped;
 $l_2(x, y), S_2(z)$ are the length and area of the perpendicular cross-section of the parallelepiped, coming into the second phase.

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NONSTATIONARY TEMPERATURE DISTRIBUTION IN
A HOLLOW CYLINDER

A. I. Logvinenko and V. N. Serebryanskii

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A relation is presented for calculating the nonstationary temperature distribution in an infinite hollow cylinder heated from the inside and cooled from the outside. To simplify engineering calculations we have obtained the values of the first three roots of the characteristic equation for the following ranges of parameters:

$$Bi_1 = 0.1 - 40; \quad Bi_2 = 0.1 - 5.0; \quad M = 0.4 - 0.9.$$

Here Bi_1 characterizes the rate of heating the wall, and Bi_2 the rate of cooling; M is the ratio of the inside to outside radius of the cylinder.

The values of the roots presented can also be used as input data for finding the corresponding roots for other values of the parameters.

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OPTIMIZATION OF HEATING "THIN" BODIES IN
THROUGH FURNACES

M. K. Kleiner

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This work is devoted to the determination of the thermal and temperature systems which minimize the total expenditure on fuel, on losses associated with waste of metal and on expenditure which depends only on the length of the furnace – capital and operational – in the case of heating of "thin" bodies in through furnaces. In this article the mathematical model of the process of heating thin bodies developed in [1] is used.

We will assume that the metal waste is proportional to the duration of heating, but its relationship with the temperature is given in a general form by the function $y(t)$; the specific expenditures depending only on the length of the furnace are equal to $C_l x$, that is, they are directly proportional to it; the specific heat of the metal and the heat exchange coefficient α do not depend on the temperature.

Minimization of the function of the total expenditures

$$I[w] = \int_0^l [C_y y(t(x)) + C_w w(x) + C_l] dx \quad (1)$$

taking into account the differential equation of connection (the thermal balance of the metal [1]) is accomplished by using the principle of the Pontryagin maximum. Solution of the problem is obtained in the form

$$w^*(x) = -KP + \sqrt{(KP)^2 + \alpha p C_{yw} y(t) + C_l}, \quad C_{yw} = C_y / C_w, \quad (2)$$

$w(x)$ is the water number of gases entering the furnace in a unit of time per unit of its length; x is the distance; K, P is the coefficient of thermal losses and the lateral perimeter of the working area; p is the surface of the heated metal arranged on a unit of length of the furnace; C_y are the losses associated with the loss of 1 kg of metal; C_w is the cost of a unit of the water number of gases; C_l is the integration constant, determined from the condition $t(l) = t_f$, l is the complete length of the furnace, t_f is the given final temperature of the metal. The optimal magnitudes are indicated by an asterisk.

Where $C_w = 0$ we find $w^*(x) \rightarrow \infty$ (2) that is, the quickest heating will be optimal. If C_l is a function of l , then (2) does not vary.

The optimal length of the furnace is determined from the relationship

$$w^*(l^*) = KP \frac{t_g^{01} - t_f}{t_g^{01} - t_f} + \left\{ \left(KP \frac{t_f - t_{cr}}{t_g^{01} - t_f} \right)^2 + (\alpha p + KP) KP \right. \\ \left. \times \frac{t_f - t_{cr}}{t_g^{01} - t_f} + \left(\alpha p + KP \frac{t_g^{01} - t_{cr}}{t_g^{01} - t_f} \right) \left[\frac{C_y}{C_w} y(t_f) + \frac{C_l}{C_w} \right] \right\}^{1/2}, \quad (3)$$

C_l is the derived specific expenditure – capital and operational – in a unit of time per unit of length of the furnace.

In the article expressions are given for the temperatures of gases and metals for the variation of $w(x)$ according to (2) as well as a method for calculating the furnace and dynamics of heating of metal in the absence of any limitations or with limitations in the supply of heat or in the temperature of the stonework. In addition, analysis of the influence of the length of the furnace on the total losses at different values of C_l is carried out, the dynamics of heating the metal is determined for a concrete example, and the influences of the allowances made are discussed.

All-Union Scientific Research Institute of the Tube Industry, Dnepropetrovsk. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 21, No. 1, p. 183, July, 1971. Original article submitted July 20, 1970; abstract submitted January 19, 1971.

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Iterative calculation (by a method of consecutive approximations) enables the variation of α and of other magnitudes during the heating process to be determined. Analysis shows that $w^*(x)$ increases during heating owing to the increase of α and $y(t)$ with the temperature.

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COMPARISON OF DISCHARGES OF FREE-FLOWING
MATERIAL FROM PLANE AND CYLINDRICAL HOPPERS

V. E. Davidson, A. P. Tolstopyat,
and N. P. Fedorin

UDC 532.529.5

For convenience in conducting experiments it is preferable to study the flow of free-flowing bodies in plane hoppers, whereas the cylindrical hoppers are used more widely in industry. A problem therefore arises concerning the mechanism of reduction of the discharge of a free-flowing material obtained in a plane hopper through an aperture of rectangular shape to a discharge from a cylindrical hopper through a circular aperture.

Experiments were carried out using a plane hopper, with a view to establishing a corresponding theory. Quartz sand was used as the free-flowing material.

In the work of F. E. Keneman a universal relationship between the discharge of a free-flowing material from a cylindrical hopper and the relative dimensions of the aperture was derived on the basis of the similarity theory

$$K(\delta) = \frac{G}{\sqrt{g} \gamma_m D^{2.5}}; \quad \delta = \frac{D}{d_e} \quad (1)$$

We compared similar (1) relationships (Fig. 1, curve, 1) obtained in a plane hopper.

The attempt to establish an agreement between the discharges through plane and cylindrical hoppers in the case of equal-sized areas of discharge apertures or in the case of equal hydraulic diameters of the latter leads to relationships which differ qualitatively from the curve 1.

The agreement established below is based on the concept of a dynamic arch which is formed above the discharge apertures. In the cylindrical hopper the dynamic arch is axisymmetrical, and compressive forces act upon the particles of the free-flowing material which pass through it in the horizontal plane. In the dynamic arch which forms in the flow from the plane hopper through the aperture which traverses the whole of the bottom of the hopper, the compressive forces are absent. If it were possible in the case of the cylindrical hopper to eliminate the presence of the above-mentioned compressive forces, then the flow process would be represented as a flow between parallel plates through an aperture whose width is equal to the diameter of the aperture of a cylindrical hopper. The discharges per unit of area of the aperture of plane and cylindrical hoppers would be the same. Allowing this and introducing the parameters K^* , which is similar to the parameter K and which represents a dimensionless discharge from a cylindrical hopper without taking into account the compressive forces, we will obtain

$$K^* = \frac{G^*}{\sqrt{g} \gamma_m b^{2.5}} = \frac{\pi \lambda G_{pl}}{4 \sqrt{g} \gamma_m b^{2.5}} \quad (2)$$

By forming the difference $K^* - K = \Delta K(\delta)$ taking into account (1), (2) and assuming $b = D(\delta^* = \delta)$, we will obtain the unknown connection between the discharges from the cylindrical and plane hoppers

$$\frac{G}{G_{pl}} = \frac{\pi}{4} \lambda \left(1 - \frac{\Delta K}{K}\right) \quad (3)$$

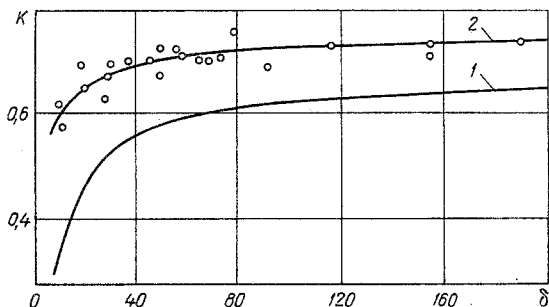


Fig. 1. Relationships $K(\delta)$ and $K^*(\delta^*)$: 1) relationship $K(\delta)$; 2) relationship $K^*(\delta^*)$.

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It is possible to divide the region $\delta \geq 200$, where the magnitudes K^* and K are self-modelling in relation to δ , in which

$$\frac{G}{G_{pl}} = 0.683\lambda.$$

NOTATION

| | |
|-------------------|---|
| a and b | are the dimensions of the aperture; |
| γ_m | is the weight of 1 m^3 of the free-flowing material; |
| G | is the weight discharge of free-flowing material from a cylindrical hopper; |
| G_{pl} | is the weight discharge of free-flowing material from a plane hopper; |
| D | is the diameter of the discharge aperture of the cylindrical hopper; |
| g | is the acceleration of gravitational forces; |
| $\lambda = b/a$; | |
| d_e | is the equivalent diameter of the particles. |